Numerical Modeling and Analysis of Electrostatic Ion Propulsion

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***Abstract*—Electrostatic ion propulsion uses two electrically-charged grids to accelerate ionized fuel atoms to very high velocities. By simplifying and analyzing this system we built a numerical model that allows us to investigate the system's behavior with a range of initial conditions.**

I. Introduction

THE idea of using electrically-charged plates or grids to accelerate charged particles as a means of providing thrust has been around since the early 1960's. However, it was not until the 1990's that electrostatic ion thrusters became a viable and accepted means of propulsion. NASA has developed several practical ion thrusters, notably the NSTAR engine which successfully powered the Deep Space 1 probe. Other designs are currently seeing use on satellites, providing maneuvering capability.[1]

A generic electrostatic ion engine is shown in Figure 1. Fuel atoms, most typically xenon, are injected from the left into the main chamber of the engine, where they are ionized and form plasma, which is contained by a magnetic field. The electrically-charged grids on the right attract and accelerate the ionized fuel atoms, extracting them from the core of the engine and accelerating them through holes in the grid and away from the spacecraft, providing thrust. Our analysis of electrostatic ion propulsion will focus on these electrically-charged grids and their interactions with the fuel atoms.

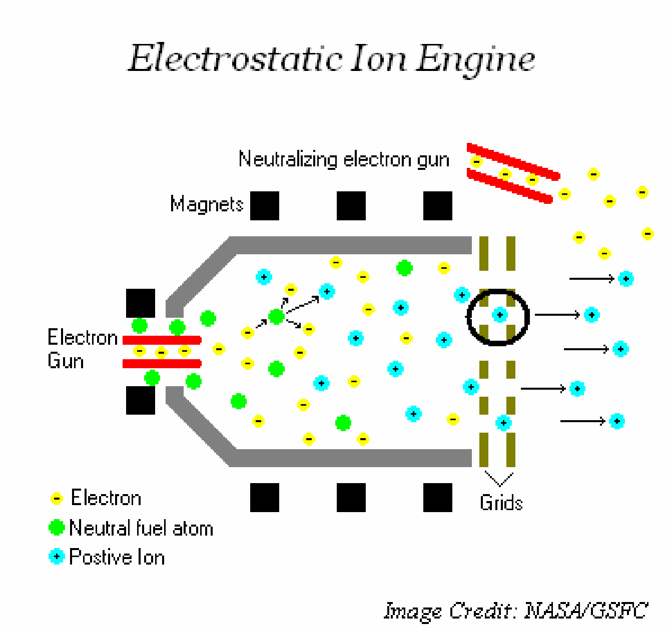

FIGURE 1 – An Electrostatic Ion Engine. This diagram shows fuel atoms injected at left, ionized, and then accelerated through two charged grids and into space, where the electrons stripped off the atoms are injected to neutralize the exhaust and the spacecraft. The black circle denotes the region we have chosen to investigate and model.

We will first seek to understand and model this pair of grids through analysis of the physical forces present and the interactions of its various components. This will provide a basic understanding of the system and the origins of behaviors it exhibits. This will also offer a base of knowledge from which to further investigate the system. From this base we will investigate the influence of the ratio of charge on one grid to the charge of the other grid, and qualitatively show how changes in this ratio result in markedly different behaviors.

We begin with a basic physical analysis of the system.

II. System Analysis

A. Variables and Assumptions

A simplified version of an ion engine, having only the two charged grids and an ionized fuel atom, is shown in Figure 2. This simplification represents a small region of the charged grids and will enable us to model on the small scale the operation of an ion engine. We have chosen to not model the region in which fuel atoms are ionized. Modeling the plasma they form and the magnetic field used to contain this plasma is complex. Also, our research indicates that the magnets are used solely to enhance the ionization of the xenon, and can thus be ignored for the study of the grid region.[2]

Our simplification involves two two-dimensional rings of outer radius *r* and inner radius *Cr*, where *C* is some constant less than 1. The two rings have surface charge density σ and are separated by distance *d*. One ring is located on the xz-plane and the other is in a parallel plane *d* units from first, with both rings centered around the *y*-axis. The ionized fuel atom is assumed to be one atom of xenon, mass *m*, ionized to the Xe^+ state, charge *q*, and located at position $\bar{r}$.

Our model involves simplifying the region around the charged rings to involve only one fuel atom and one grid hole. We ignore all atomic interactions prior to reaching the grid and the electric field generated by the rest of the grid. As such the only force involved is that between the electric field of the two rings and the ionized atom.

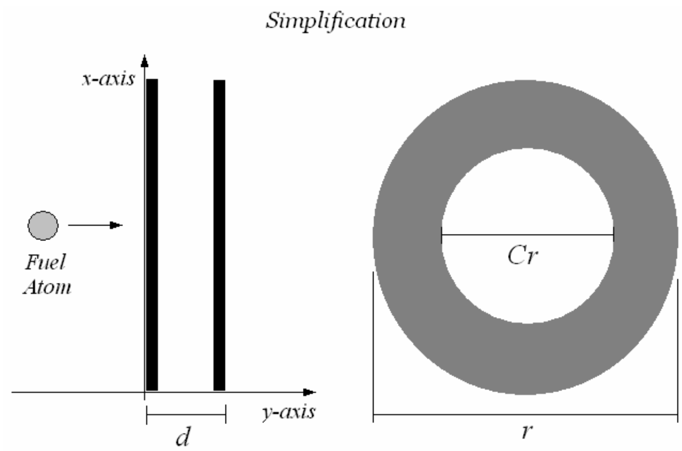

FIGURE 2 – Diagram of Simplified Electrostatic Ion Engine with Variables and Coordinate System. The region circled in Figure 1 is simplified above for our model. Two charged rings, representing a small section of the grid shown in Figure 1, accelerate one fuel atom through their center and out into space.

B. *Governing Equations*

The equation for an electric field created by a surface is

$$\vec{E}(\bar{r}) = \int_{Surface} \frac{k\sigma(\bar{r} - \bar{r}')}{\left|\bar{r} - \bar{r}'\right|^3} dS \quad (1)$$

where σ is the charge density of the surface, $\bar{r}'$ is the location of differential surface *dS*, and k is the electrostatic proportionality constant $(4\pi\varepsilon_0)^{-1}$.

For the first ring we evaluate this integral over the surface of the ring from the inner radius *Cr* to the outer radius *r*, substituting in for $\vec{r}$ and $\vec{r}'$

$$\vec{E}(\bar{r}) = \int_0^{2\pi}\int_{Cr}^{r} \frac{k\sigma\left[(x - r\cos\theta)\hat{i} + y\hat{j} + (z - r\sin\theta)\hat{k}\right]}{\left[(x - r\cos\theta)^2 + (z - r\sin\theta)^2 + y^2\right]^{\frac{3}{2}}} rdrd\theta \quad (2)$$

Equation 2 gives the field for one plate. By the Law of Superposition,

$$\vec{E}_{Total}(\bar{r}) = \sum \vec{E}_{plate} = \vec{E}_{r_1} + \vec{E}_{r_2} \quad (3)$$

We can substitute values for the second ring into Equation 2 to give the field produced by the second ring. The sum of these two integrals would then be the total electric field. An analytical solution can be found along the y axis by simplifying Equation 2

$$\vec{E}(y\hat{j}) = \int_0^{2\pi}\int_{Cr}^{r} \frac{k\sigma\left[(-r\cos\theta)\hat{i} + y\hat{j} + (-r\sin\theta)\hat{k}\right]}{\left[r^2 + y^2\right]^{\frac{3}{2}}} rdrd\theta \quad (4)$$

$$\vec{E}(y\hat{j}) = \frac{k\sigma 2\pi z}{\sqrt{r^2 + y^2}} - \frac{k\sigma 2\pi z}{\sqrt{(Cr)^2 + y^2}} \quad (5)$$

By Newton's Law and the Lorentz Force Equation for purely electric fields

$$\vec{F} = q\vec{E} = m\vec{a} \quad (6)$$

the acceleration of the atom can be determined for any position $\bar{r}$. From this, given initial position and velocity conditions, the trajectory of the atom through the electric field of the rings can be determined.

III. NUMERICAL MODEL

The electric field equation derived for the charged rings was implemented as a function in Matlab. We used the ordinary differential equation solver *ode45* to numerically evaluate this function. Using initial conditions of all the variables, in this case position and velocity vectors, as inputs, this model returns the time evolution of the system's motion, allowing both qualitative and quantitative analysis.

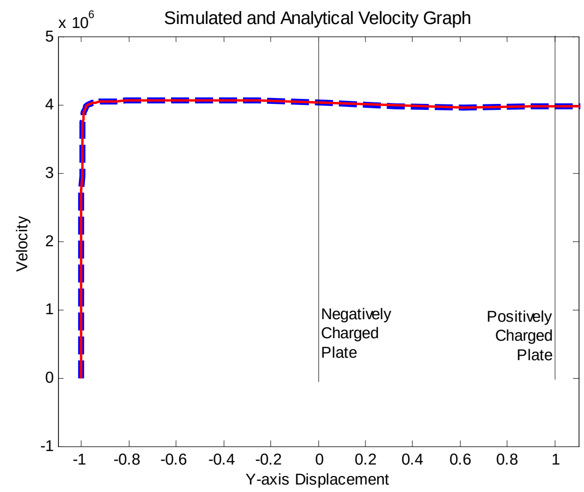

FIGURE 3 – Particle Analytical and Numerical Velocity versus Y-Axis Displacement. Red line shows numerical (i.e. simulation) solution and thicker blue line shows the analytical solution. The ion was given no initial velocity and the plates were given equal and opposite charges.

IV. RESULTS

We used our Matlab simulation to qualitatively investigate the nature of the electrostatic field generated by the two charged rings. We used a variety of different initial position and velocity conditions for the particle along with several different ring charge configurations to examine the influence of each condition on the motion of the Xe^+ atom. Figure 3 shows the analytical and numerical calculations for the velocity of the atom plotted against the atom's position. The two methods give identical results, showing that our simulation reflects the theoretical motion of the particle (a Xe^+ atom), through two electrically charged rings.

The behavior of the atom due to the electric field is dependent on the charges of the two plates. For our model we started the particle closest to the negative ring, so the negative ring extracts the particle from the engine, and then the combined force of the two rings accelerates it. When the magnitude of charge on the positive plate is much greater than that of the negative plate, the particle behaves as Figure 4a shows. The positively-charged particle is repelled by the very positively-charged plate. The particle is immediately accelerated back into the engine.

When the two plates are of equal charge magnitude, the particle is attracted to the negatively-charged plate and accelerates through it as shown in Figure 4b. Interaction with the opposite fields produced by the two rings causes the particle to oscillate back and forth between the plates before eventually being ejected back into the engine.

With the negative plate having a charge of greater magnitude than the positive plate, the particle is extracted from the engine and again exhibits the oscillating behavior between the two plates until it is finally ejected away from the engine. This can be seen in Figure 4c. The negative plate must have a greater charge than the positively charged plate in order for the acceleration of the particle to be in the correct direction.

However, through experimentation with different surface charges on the two grids, we found that the only way to successfully extract a positive ion from the engine was using negative grid on the engine side of the pair and a positive grid on the exhaust side. Further research revealed this is opposite of the geometry used in real world engines. [2]

This means that the engine side grid must have a positive charge, and thus to get a positive ion through it, another force is required. Based on this we now believe, against our research, that the magnets surrounding the engine chamber are not only responsible for improving ionization of the fuel but also for doing work against the field of the positive grid until the atom passes through the grid and can be accelerated away from the engine. No other system exists on the engine that could be capable of overcoming the powerful electric field on the grids.

V. Conclusion

Our simulation and investigation has taken a major first step towards accurately modeling an electrostatic ion engine. Our model succeeds in reproducing the interaction of a particle with the grids in isolation. A true model of the operation of an ion engine will incorporate this system but also include the engine's other systems and forces. Modeling the plasma and the magnetic field region is an essential next step to creating an accurate and effective model of the system. With time a very accurate mathematical model should be possible.

Because electrostatic ion propulsion is a new technology there is much room for optimization and innovation. Future directions of research could begin by confirming model results with physical experiments. Further study could investigate more efficient propellant mixtures or optimize ionization methods. An optimization of the grids' charges is another obvious course to take. A more extensive simulation that showed the ionization of the atoms and the effects of the

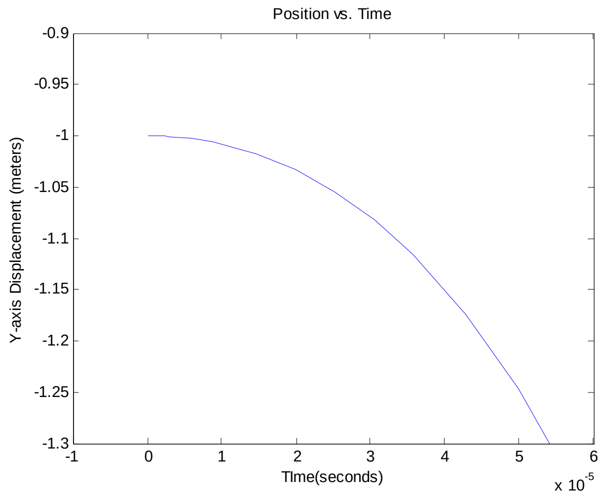

FIGURE 4a- Xenon Ion Displacement v. Time when the positively charged plate is greater than the negatively charged plate. The atom is simply repelled by the large positive field. The negatively charged plate is at y=0 and the positively charged plate is at y=1.

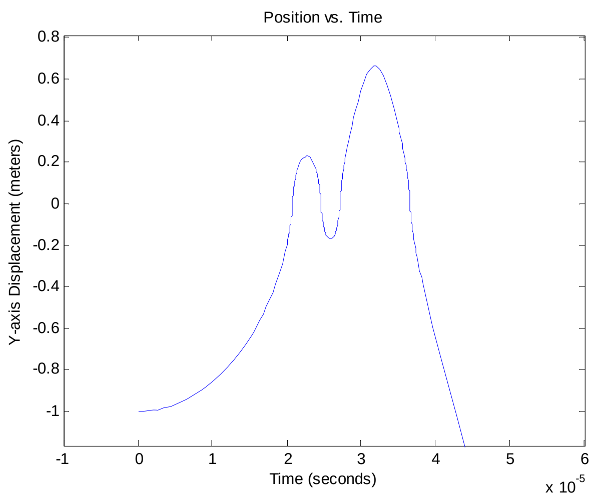

FIGURE 4b- Xenon Ion Displacement v. Time when the magnitudes of the positively and negatively charged plates are equal. The atom oscillates in and out of the rings before being accelerated back into the engine.

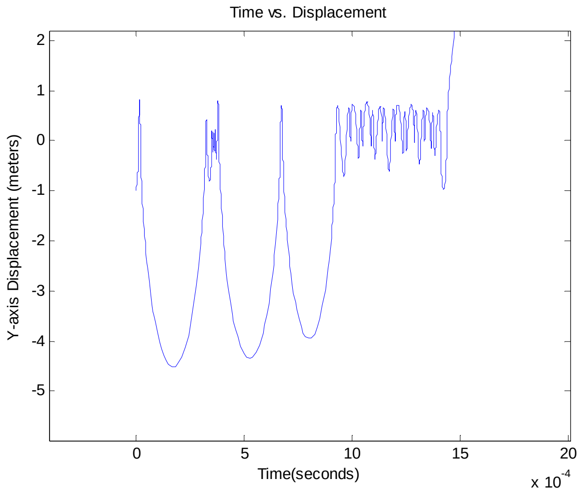

FIGURE 4c- Xenon Ion Displacement v. Time when the negatively-charged plate is stronger than the positively charged plate. The atom oscillates in and out of the rings before being accelerated out of the engine.

interactions of the charged atoms would aid in finding more ways to improve the engine. Some research suggests that an additional pair of grids may improve the efficiency of the engine; the model could be used to explore this and other concepts. An improved version of our simulation enables a wide range of possible investigations and could lead to improvements in space propulsion methods.

ACKNOWLEDGMENT

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[2] NASA Glenn Research Center Deep Space-1 History Site. http://www.nasa.gov/centers/glenn/about/history/ds1.html .