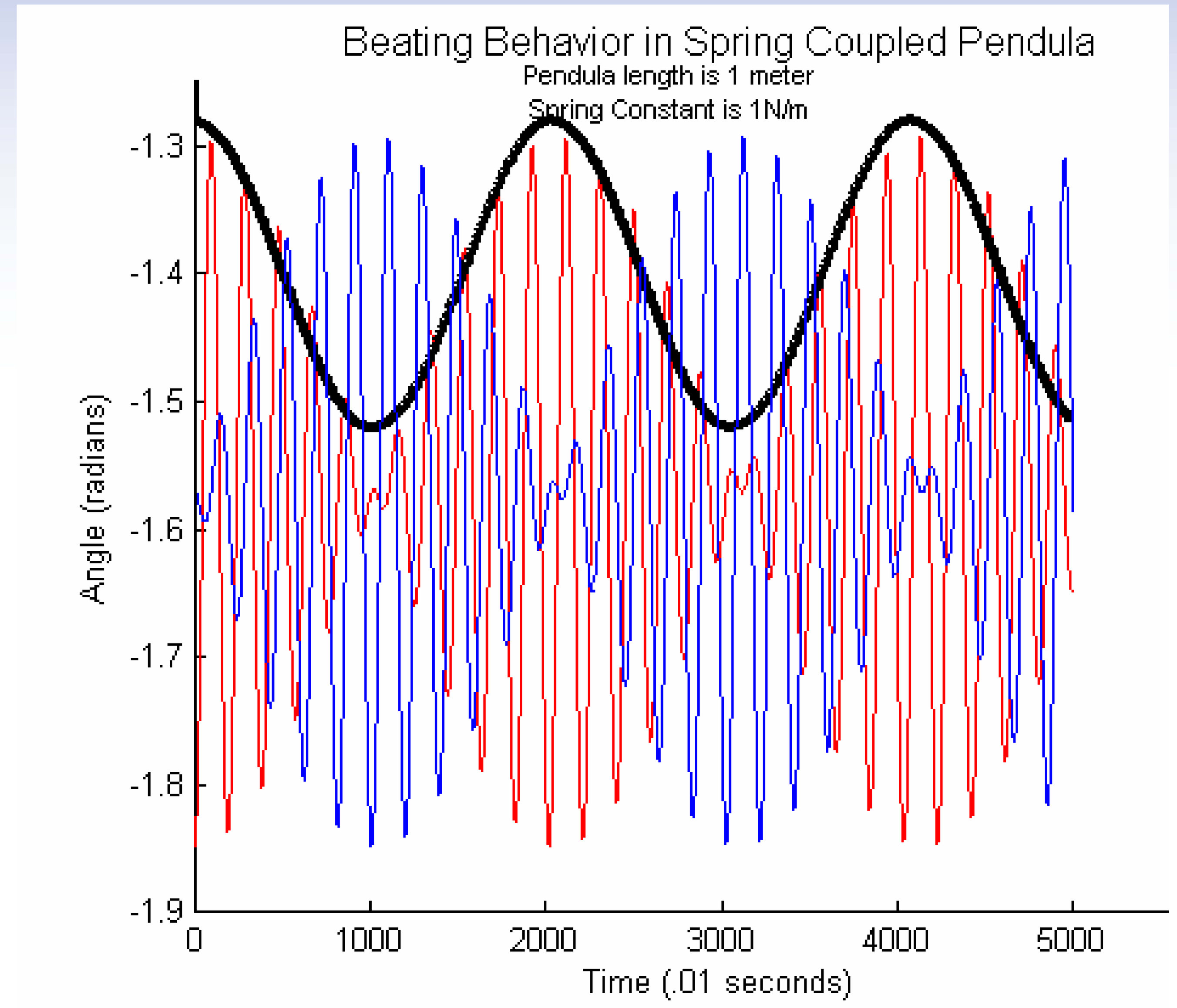
Beating Behavior in Spring-Coupled Pendula

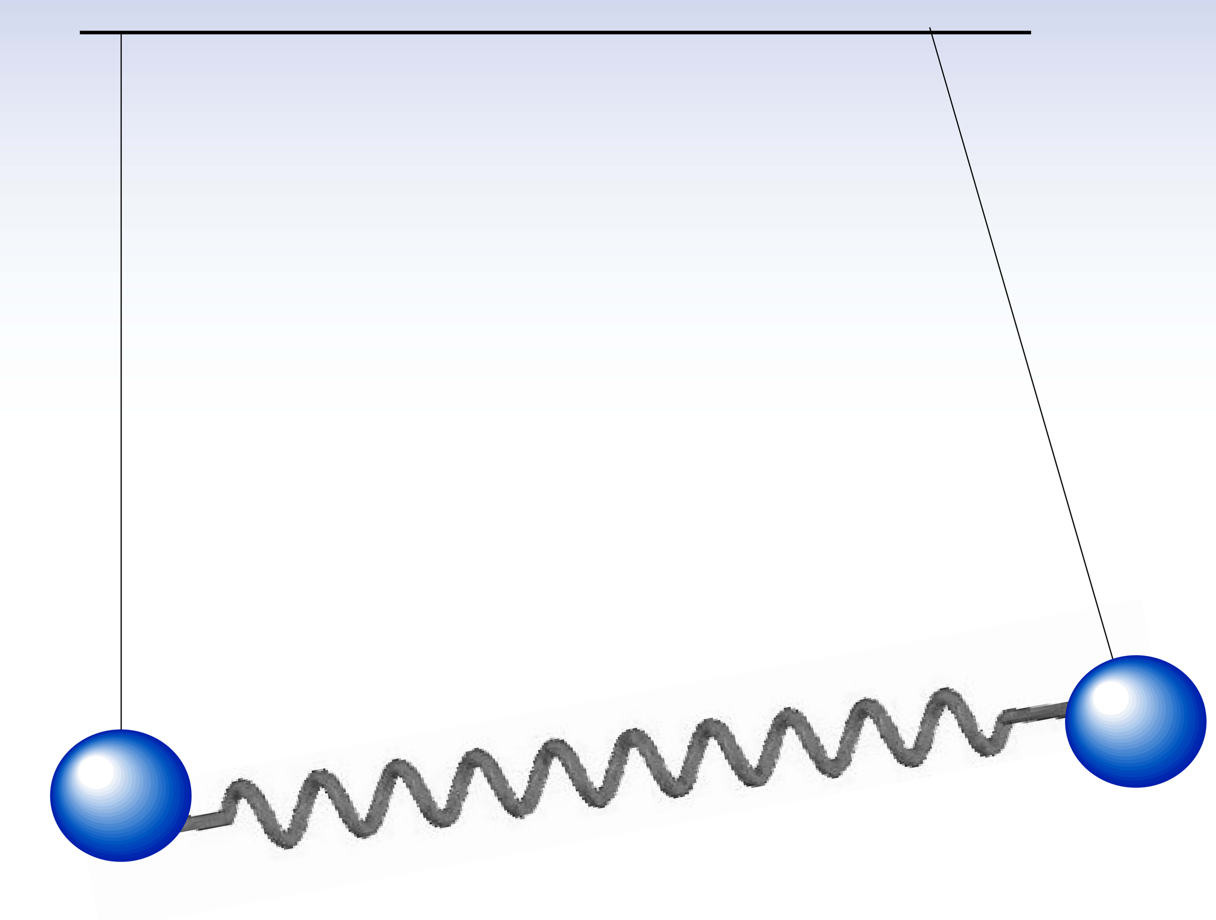
Sarah Zimmermann and David Stamp

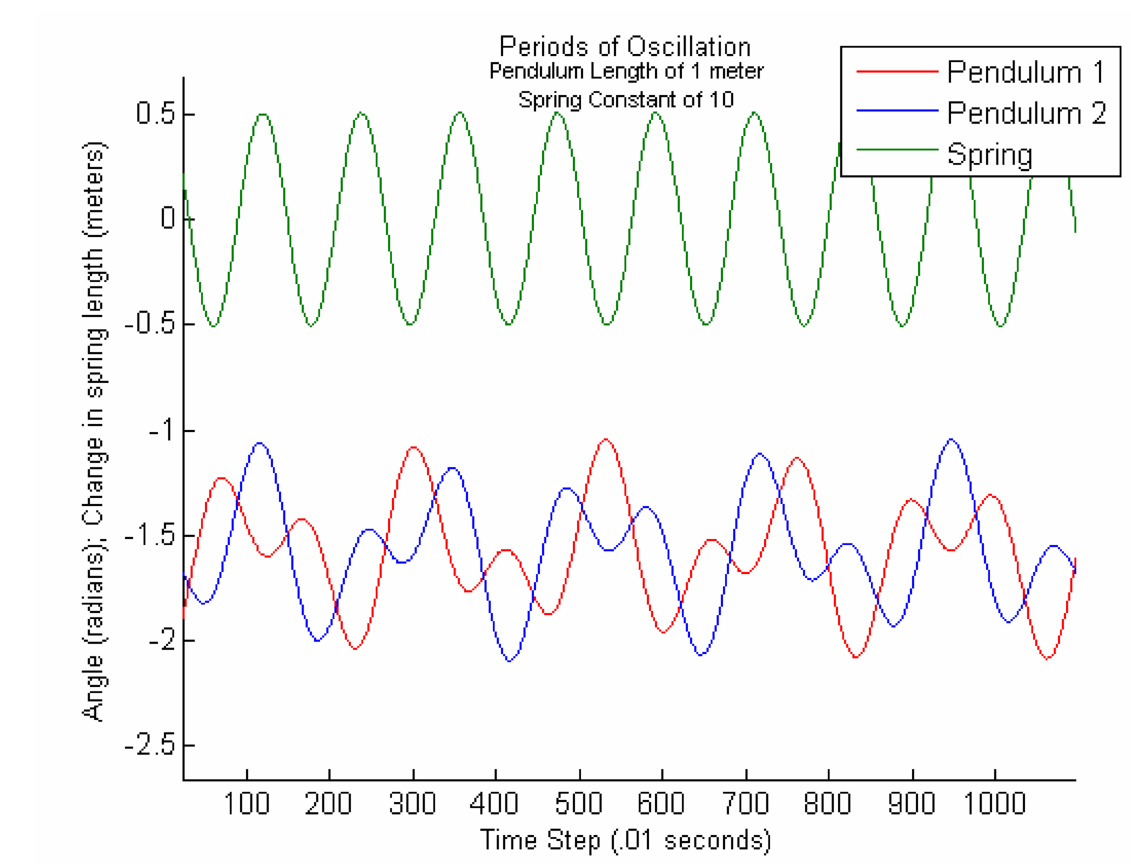
Abstract

We selected the system of spring-coupled pendula seen to the right and derived governing equations that allow us to compute the evolution of our system over time. After implementing these equations in Matlab, we noticed that the system exhibited “beating” behavior; the sinusoidal shape of each pendulum’s position over time plot as energy is transferred between the two pendulums. We used our mathematical model to investigate the influence of different initial conditions on the properties of the resultant sine wave.

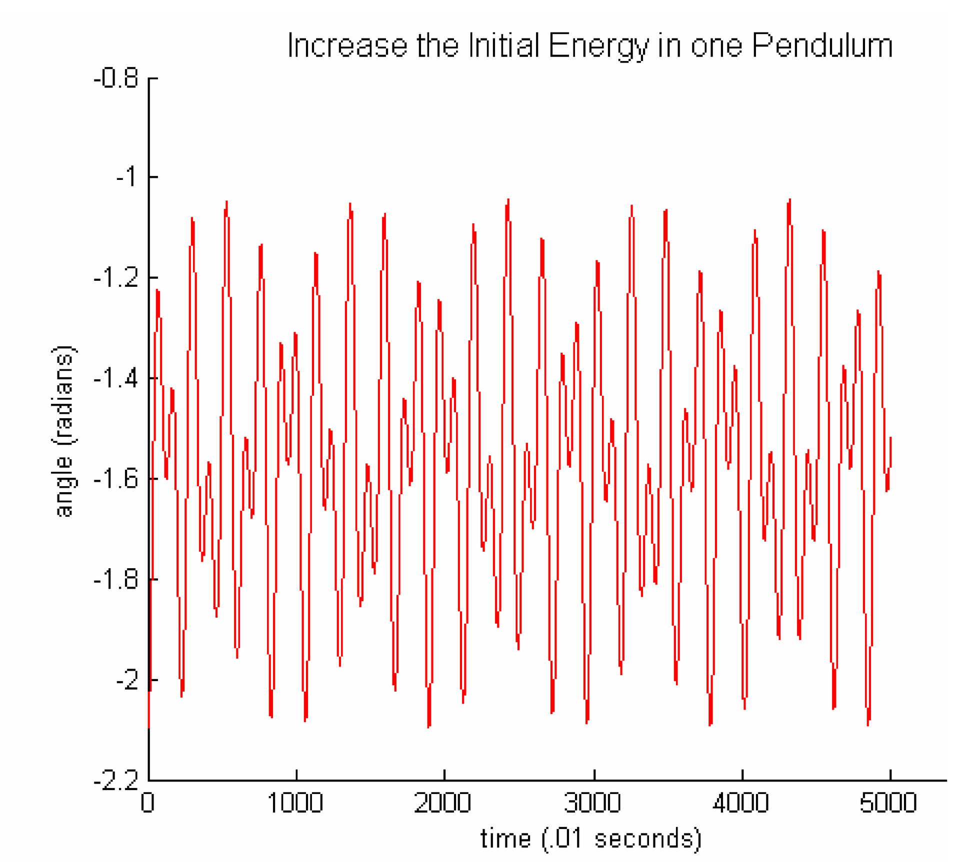


Beating Behavior of the pendulums. The two pendulums transfer energy evenly from pendulum to the other in a Sinusoidal manner. The change in the amplitude of the period of oscillation of the pendulum is a sine wave. This image shows the beating caused by two pendulums with a pendulum length of one meter and a spring with a spring Constant of 1 N/m.





Periods of oscillation of a Spring Coupled System. The pendula are perfectly out of phase with each other. The position of one pendulum is determined by the oscillation of one pendulum as well as the oscillation of the spring. The oscillation of the pendulum is a summation of the oscillations of the spring and the other pendulum.



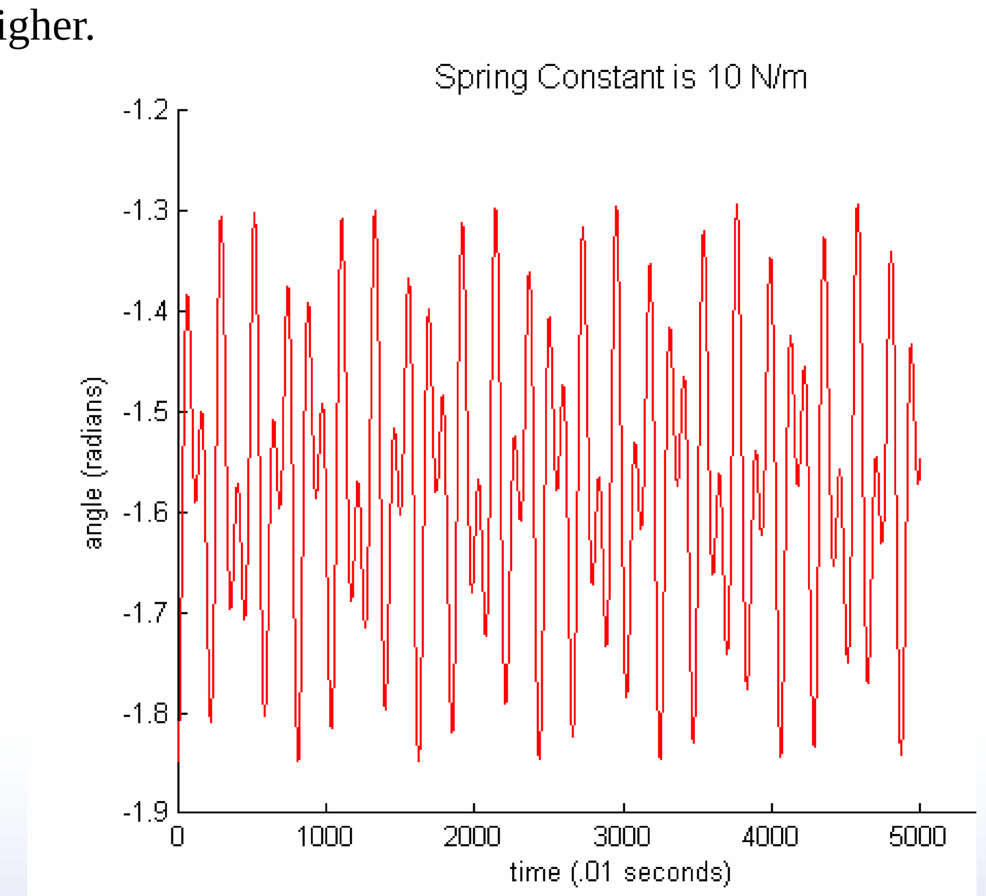
Increasing the initial energy in one pendulum. When the initial energy of one pendulum is increased by adding potential energy and raising the pendulum higher, the amplitude of the phase gets higher.

General Periodic Equation

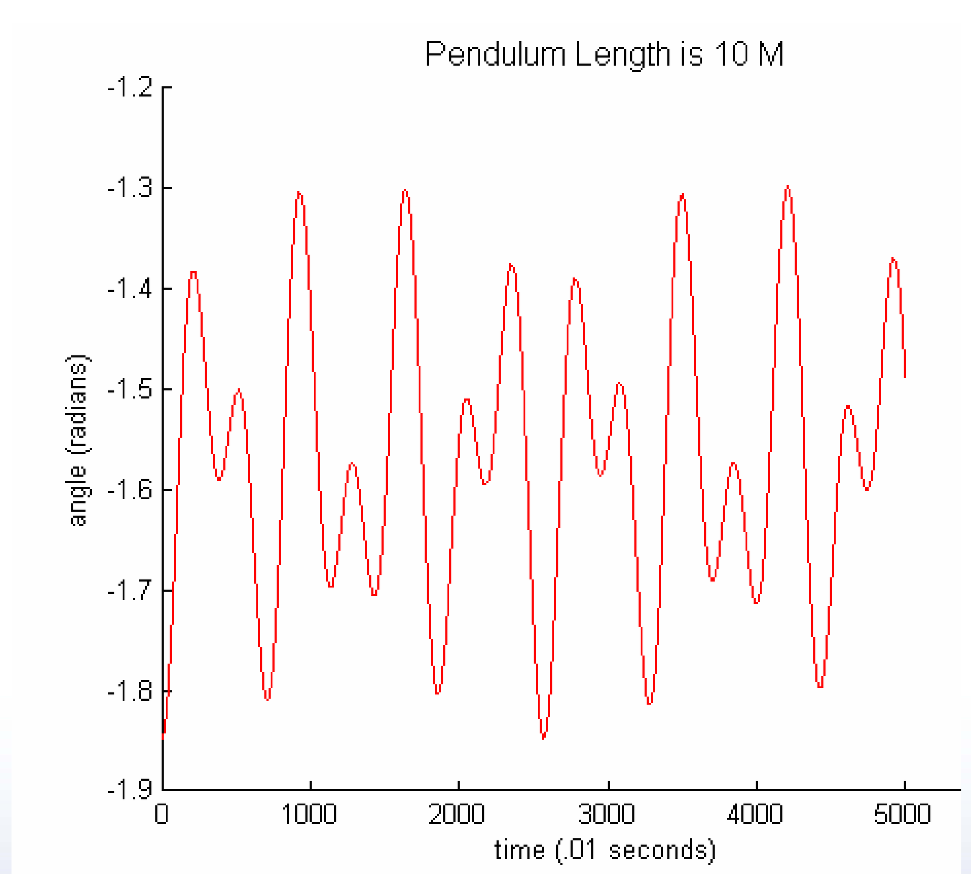
$$T = 2\pi\sqrt{\frac{l}{g}}$$

General Wave Formation Equation

$$Asin(\omega t) + d$$



Results of an increase in the Spring Constant. When the spring constant is increased, the period of the change in amplitude of the period of the pendulum decreases. The beating occurs more rapidly as the spring has more force. In this image, the spring constant is increased by a factor of 10.



When the length of the pendula are changed, the period of oscillation changes. This plot shows the period of a pendulum with a length of 10 meters. Compared to the pendulum with a length of 1 meter, the period of osculation is increased according to the equation for the period of a spring.

Conclusion

The qualitative data we collected from our Matlab simulation describes how different initial quantities influence the properties of the resulting sine wave, described by the equation above right. We also discovered that the frequency of each pendulum’s oscillations is modeled almost exactly by the equation for the period of a free-standing pendulum, above left. Research into this beating phenomenon revealed, however, that in order to mathematically describe the sine wave or exactly how different variations influence it requires mathematical techniques to which we have not been exposed, and will thus have to wait for a future investigation.